

W8 Final

1. $3 \quad 11 \quad 19 \quad 27 \quad 35$
 $\swarrow \quad \swarrow \quad \swarrow \quad \swarrow$
 $8 \quad 8 \quad 8 \quad 8$

This is an arithmetic sequence

$$a_n = a_1 + (n-1)d$$

$$a_n = 3 + 8(n-1)$$

$$= 3 + 8n - 8$$

$$a_n = 8n - 5$$

B.

2
$$a_n = (-1)^n \left(\frac{n}{n+11} \right)$$

$$a_1 = (-1)^1 \left(\frac{1}{1+11} \right) = -\frac{1}{12}$$

$$a_2 = (-1)^2 \left(\frac{2}{2+11} \right) = \frac{2}{13}$$

$$a_3 = (-1)^3 \left(\frac{3}{3+11} \right) = -\frac{3}{14}$$

$$a_4 = (-1)^4 \left(\frac{4}{4+11} \right) = \frac{4}{15}$$

$$a_5 = (-1)^5 \left(\frac{5}{5+11} \right) = -\frac{5}{16}$$

$$-\frac{1}{12}, \frac{2}{13}, -\frac{3}{14}, \frac{4}{15}, -\frac{5}{16}$$

E

3.

$$S = \frac{a_1}{1-r} \quad a_1 = 6$$

$$r = \frac{6}{7}$$

$$= \frac{6}{1 - \frac{6}{7}} = \frac{6}{\frac{1}{7}} = 6 \times 7 = 42$$

E

$$4. a_1 = 0, d = -\frac{4}{5}$$

$$a_n = a_1 + (n-1)d$$

$$= 0 + (n-1)\left(-\frac{4}{5}\right)$$

$$= -\frac{4}{5}(n-1) = -\frac{4}{5}n + \frac{4}{5}$$

C

$$5. a_1 = 4, r = \frac{1}{6}, n = 10$$

$$a_n = a_1 r^{n-1}$$

$$= 4\left(\frac{1}{6}\right)^{n-1}$$

$$a_{10} = 4\left(\frac{1}{6}\right)^{10-1} = \frac{1}{2519424}$$

B

$$6. \text{Total no. of possible ways} = 11 \times 10 \times 9$$

for 1st, 2nd, and 3rd

$$= 990 \text{ ways}$$

E

7. Possible combinations

$$1^{\text{st}} = 30$$

$$2^{\text{nd}} = 30$$

$$3^{\text{rd}} = 30$$

$$= 30 \times 30 \times 30 = 27000$$

B

8. Possible outcomes

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$$P(\text{sum is less than 7}) = \frac{15}{36} = \frac{5}{12}$$

C

9. $P(\text{Attended seminar and passed exam}) = \frac{18}{75} = \frac{6}{25}$

D.

10. Length of an arc = $2\pi r \frac{\theta}{360}$

$$s = 2\pi(25) \left(\frac{75}{360} \right) = 32.7249$$

$$\approx 32.72 \text{ cm}$$

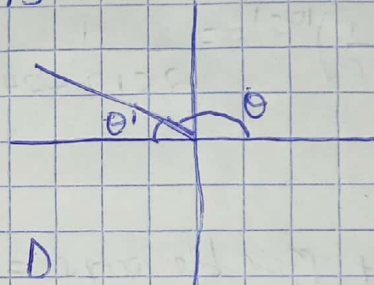
C

11.

$\theta = 165^\circ$ II quadrant

Reference angle = $180^\circ - \theta = 180^\circ - 165^\circ = 15^\circ$

$\theta' = 15^\circ$



D

12. Area of a sector = $\frac{1}{2} r^2 \theta$

$$= \frac{1}{2} (5)^2 \left(\frac{\pi}{5} \right)$$

$$\approx 20.11 \text{ square inches}$$

E

13. $4 \cos x + 2 = 0$

$$4 \cos x = -2$$

$$\cos x = -\frac{2}{4} = -\frac{1}{2}$$

$$x = \cos^{-1} \left(-\frac{1}{2} \right) = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$x = \frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n$$

B

$$14. \sin \theta = \frac{\text{Opp}}{\text{Hyp}} = \frac{8}{17}$$

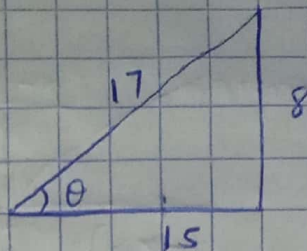
$$\cos \theta = \frac{\text{Adj}}{\text{Hyp}} = \frac{15}{17}$$

$$\tan \theta = \frac{\text{Opp}}{\text{Adj}} = \frac{8}{15}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{17}{8}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{17}{15}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{15}{8}$$



D

15.

$$180^\circ = \pi \text{ radian}$$

$$-30 \times \frac{\pi}{180} = -\frac{\pi}{6}$$

C

$$16. (3, 4) \quad x - y = 11 \quad x - y - 11 = 0$$

$$(a, b) \quad Ax + By + C = 0$$

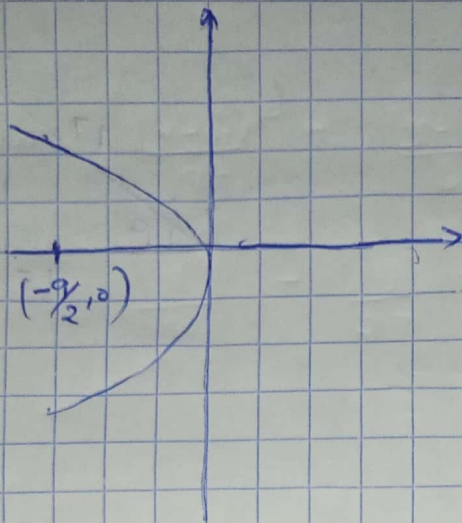
$$d = \frac{|Aa + Bb + C|}{\sqrt{A^2 + B^2}} = \frac{|3 - 4 - 11|}{\sqrt{1 + 1}} = \frac{|-12|}{\sqrt{2}}$$

$$= \frac{12}{\sqrt{2}}$$

$$\approx 8.4853$$

E

17



$$y^2 = 4px \quad \text{Focus} = (p, 0)$$

$$p = -\frac{9}{2}$$

$$y^2 = 4\left(-\frac{9}{2}\right)x$$

$$y^2 = -18x$$

B

18 Foci: $(0, 0), (0, 10)$

Major axis length = 16

Centre = 16

Centre = $\left(\frac{0+0}{2}, \frac{0+10}{2}\right) = (0, 5) = (h, k)$

E

$2a = 16$

$a = 8$

$b^2 = 2c = \sqrt{(0-0)^2 + (0-10)^2}$

$2c = \sqrt{0+100} = 10$

$c = 5$

$c^2 = a^2 - b^2$

$b^2 = a^2 - c^2$

$= 8^2 - 5^2$

$b = \sqrt{64-25} = \sqrt{39}$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$= \frac{x^2}{39} + \frac{(y-5)^2}{64} = 1$

A C

19. Vertices: $(\pm 3, 0) = (h \pm a, k)$

foci: $(\pm 6, 0) = (h \pm c, k)$

Centre: $(0, 0) = (h, k)$

$a = 3, c = 6$

$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2 = 6^2 - 3^2$

$b = \sqrt{36-9} = \sqrt{27}$

Equation: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$$\frac{x^2}{9} - \frac{y^2}{27} = 1$$

C

20. $3x - 3y = 3$
 $9x + 8y = 11$

$$\begin{aligned} 3x - 3y &= 3 \\ 3y &= 3x - 3 \\ y &= x - 1 \end{aligned}$$

$$m_1 = 1$$

$$\begin{aligned} 9x + 8y &= 11 \\ 8y &= -9x + 11 \\ y &= -\frac{9}{8}x + \frac{11}{8} \end{aligned}$$

$$m_2 = -\frac{9}{8}$$

$$\begin{aligned} \tan \theta &= \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right| \\ &= \left| \frac{-\frac{9}{8} - 1}{1 + (-\frac{9}{8})} \right| \end{aligned}$$

$$\tan \theta = |17|$$

$$\theta' = \tan^{-1}(17) = 1.512 \text{ radians} \approx 86.6^\circ$$

A